Math 105, Assignment 4 **Due 2015-03-16, 4:00 pm**

In the following problems you are expected to justify your answers unless stated otherwise. Answers without any explanation will be given a mark of zero. The assignment needs to be in my hand before I leave the lecture room or you will be given a zero on the assignment! **Don't forget to staple your assignment! You may lose a mark if you do not.**

1. Evaluate the following:

   (a) $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\cos(2x)\sin^5(x)}{\tan^2 x}dx$

   (b) $\displaystyle\int_0^{\pi/4} \tan^5(x)dx$

   (c) $\displaystyle\int \frac{y}{(y^2+4y+8)^{3/2}}dy$

   (d) $\displaystyle\int \frac{t^4+3t^3+6}{t^2+4t+3}dt$

   (e) $\displaystyle\int \frac{\cos x}{(\sin x-1)^2(\sin x+1)}dx$ **Hint:** Let $u=\sin x$

   (f) $\displaystyle\int \frac{x}{x^2+6x+18}dx$

2. Evaluate the following improper integrals:

   (a) $\displaystyle\int_0^a \frac{1}{\sqrt{a^2-x^2}}dx, \quad a>0$

   (b) $\displaystyle\int_1^\infty \frac{\log x}{x^{3/2}}dx$

   (c) $\int_0^\infty xe^{-x^2}e^{\int_0^{x^2} e^{-t}dt}dx$

   Hint: Let $u=\int_0^{x^2} e^{-t}dt$ and be careful of the limits of integration.

3. (a) Find the approximation using 6 rectangles ($n=6$) of

   $$\int_0^1 e^{x^2}dx$$

   by using:

   i. Midpoint rule

   ii. Trapezoid rule

   iii. Simpson's rule

   (b) Given that the fourth derivative of $f(x)=e^{x^2}$ satisfies

   $$|f^{(4)}(x)| \le 76e,$$

   for all $x\in[0,1]$, find an upper bound on the absolute error in your approximation for Simpson's rule.

4. Solve the following initial value problems.

   (a) $\frac{dy}{dx} = xe^{x^2 - 2\log y}, \quad y(0) = 0$

   (b) $\frac{dP}{dt} = kP(N - P), \quad P(0) = 1$